

Introduction to Phase (Horizontal) Shift

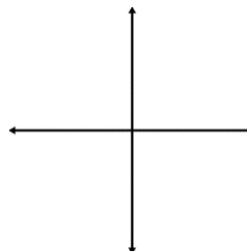
Print these 3 pages if you can. The first 2 pages are discussed in the video, page 3 contains a couple problems for you to try. Video – Digital Learning Lesson 2 (a link to video can be found on my website)

Learning Targets:

- I can determine the Phase Shift of the Sine and Cosine functions
- I can graph transformations of the Sine and Cosine functions

Recall graphing a function like $f(x) = (x - 3)^2$.

We know the expression in the parenthesis will shift our parent graph horizontally, in this case 3 to the right.



We often think about shifting the parent graph “the opposite way”. Another way to determine the shift (and more accurately) is to “re-center” the function by solving the expression in the parenthesis to equal 0. In this case, solving $x - 3 = 0$, for $x = 3$. In the case of this quadratic function, the line of symmetry was shifted to $x = +3$

A “**phase shift**” for a trigonometric function is a *horizontal shift* of the graph. It works the same way. However, given the form of trigonometric functions, $f(x) = a \sin(bx - c) + k$, with “b” typically written inside the parenthesis, we cannot simply think of shifting the “opposite way”. We must solve the expression in the parenthesis equal to 0. The formula for phase shift is $x = \frac{c}{b}$.

1) Identify the phase shift for each.

- a) $y = 3 \sin(2x - \pi) + 4$ b) $f(x) = -\cos(3x + 2)$ c) $g(x) = \sin\left(\frac{\pi}{2}x - 3\pi\right)$

2) Identify the amplitude, period, equation of the midline and phase shift in each of the following:

	Amplitude	Period	Equation of the Midline	Phase Shift
$h(x) = \frac{3}{4} \sin(3x + \pi) - 1$				
$f(x) = -3 \cos\left(\frac{\pi}{4}x - \pi\right)$				

Sketching the graphs of trig functions that include a phase shift isn't always easy, especially when other transformations are involved. To do so, we are going to pay close attention to the coordinates of a "first point", "last point" and spacing as we plot our points along the midline. Lets graph a couple simpler equations today. In our next lesson we will graph some tougher one.

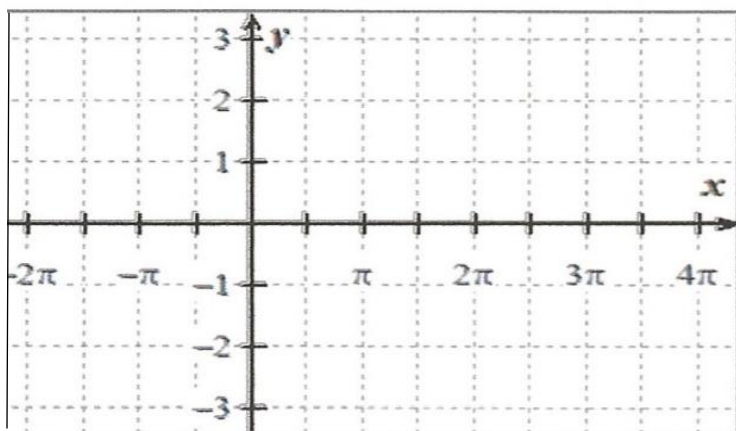
3) Sketch a graph of $y = 2\sin\left(x - \frac{3\pi}{2}\right)$ over the interval $[-2\pi, 4\pi]$

Amplitude:

Period:

Phase Shift:

Spacing:



First point:

Last point

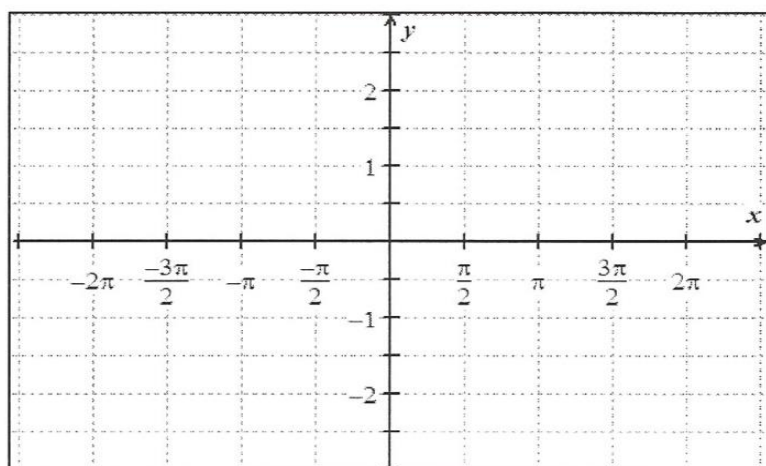
4) Sketch a graph $f(x) = \cos(2x + \pi)$ over the interval $[-2\pi, 2\pi]$

Amplitude:

Period:

Phase Shift:

Spacing:



First point:

Last point

Pause the video for a few minutes and try questions 5 and 6, then resume to check your results.

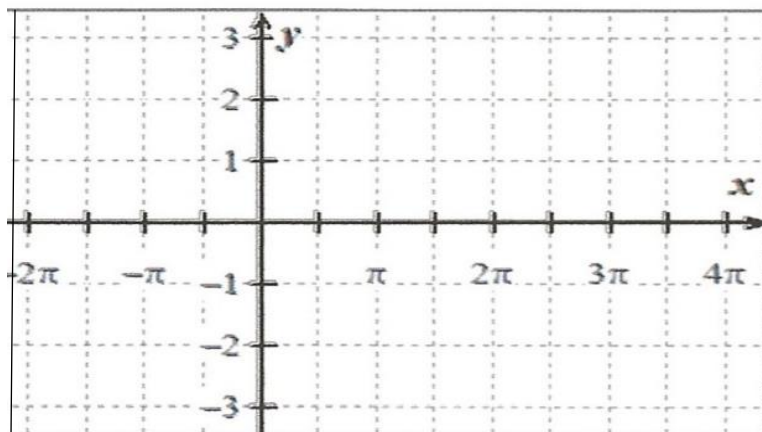
5) a) Sketch a graph of $g(x) = 3\sin\left(x - \frac{\pi}{2}\right)$ over the interval $[-2\pi, 4\pi]$

Amplitude:

Period:

Phase Shift:

Spacing:



First point:

Last point

b) Identify the coordinates of all relative extremes included in the graph

Minimum(s):

Maximum(s):

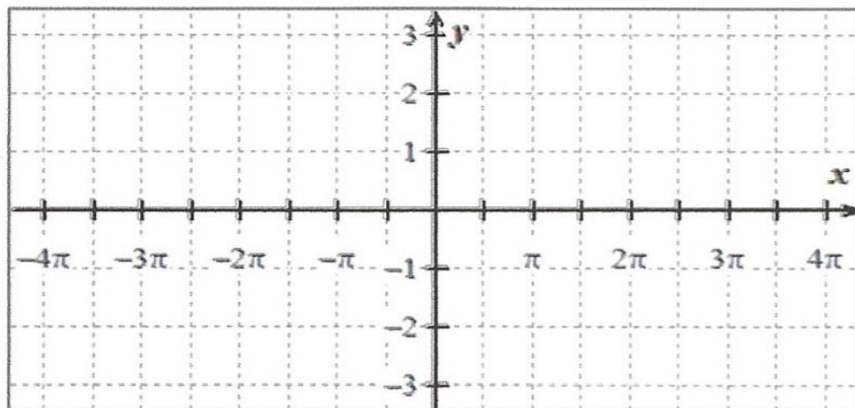
6) a) Sketch a graph $y = \cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$ over the interval $[-4\pi, 4\pi]$

Amplitude:

Period:

Phase Shift:

Spacing:



First point:

Last point

b) Identify the coordinates of all zeros included in the graph.